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# Letter to the Editor

## Reply to Comments by K.P. Soldatos on the "Accurate Equations of Laminated Composite Deep Thick Shells", *Zat. J. Solids Structures* Vol. 36, No. 19, pp. 2917–2941 (1999) by M.S. Qatu

The author received with great interest the comments made by Soldatos on the recent article by the author (Qatu, 1999). In this recent article, accurate equations for laminated composite *deep* thick shells were presented. The theory used was a first-order shear deformation theory (FSDT), where a shear correction factor was needed. An exact integration of the $(1+z/R)$ terms was made. These equations were used successfully to predict natural frequencies of laminated composite deep and thick spherical and cylindrical shells. They showed better accuracy than higher order shell theories (HSDT).

The comments made by Soldatos discussed the following "classes" of theories:

1. Two-dimensional FSDT theories (e.g. Qatu, 1999)
2. Two-dimensional HSDT theories (e.g. Reddy and Liu, 1985)
3. Three-dimensional *shallow* shell theories (e.g. Bhimaraddi, 1991)
4. Three-dimensional *deep* shell theories (e.g. Ye and Soldatos, 1994).

The statements made by Soldatos on the lack of accuracy of the equation by Bhimaraddi for deep laminated composite shells are correct. This is because Bhimaraddi made the following assumption (Eq. (2), Bhimaraddi, 1991)

$$R_1(R+z)=1$$

where $R_1$ is the principal radius of curvature. This assumption was needed to introduce constant coefficients to the partial deferential equations, which made the analysis easier. The above assumption reduced the theory to that of shallow shells. In fact, to Bhimaraddi's credit, he mentioned this limitation. However, the results he presented for $R/a = 1$ were arguably outside the applicability of shallow shell theories.

As was noted by Soldatos, the equations developed by Qatu (1999) for laminated composite deep thick shells yielded results that are extremely close to those predicted by Ye and Soldatos (1994). In the later, a three-dimensional elasticity theory was used. Knowing that the equations of Qatu are indeed two-dimensional and involve a first order shear deformation theory, shows the importance of integrating the term $1+z/R$ in the stress resultant equations. In other words, a first order shear deformation theory with exact resultants (Qatu, 1999) is more accurate than a higher order theory with approximated resultants (Bhimaraddi, 1991; Reddy and Liu, 1985) for deep shells. Table 1 shows the percentage difference between the natural frequency parameters obtained using the equations by Qatu (1999) and those by Ye and Soldatos (1994).

Table 1 answers partially the question raised by Soldatos on the need for developing a higher order shear deformation theory with exact stress resultants in the sense made by Qatu (1999). For thickness


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Table 1
Nondimensional frequency parameters $\Omega$ for [0, 90] cylindrical shells ($E_1/E_2 = 25$, $G_{12}/E_2 = 0.5$, $G_{13}/E_2 = 0.5$, $G_{23}/E_2 = 0.2$, $\nu_{12} = 0.3$, $k^2 = 5/6$, $a/b = 1$, $a/h = 10$)

| $R/a$ | Ye and Soldatos (1994) (3D) | Qatu (1999) (FSDT) | Difference (%) |
|---|---|---|---|
| 1 | 10.6973 | 10.667 | 0.28 |
| 2 | 9.4951 | 9.4428 | 0.55 |
| 4 | 9.1155 | 9.0731 | 0.47 |
| 5 | 9.0616 | 9.0221 | 0.44 |
| 10 | 8.9778 | 8.9446 | 0.37 |
| 20 | 8.9477 | 8.9194 | 0.32 |
| Plate | 8.9248 | 8.9001 | 0.28 |

ratios of 0.1, there is no need. For higher thickness ratios, 0.2 or higher, the need is yet to be established.

The degree of applicability of the equations presented by Qatu (1999) for different and complex shell geometries need to be tested. It is true that FSDT approximate various parameters with the shear correction factors. It is also true that HSDT introduce boundary terms that are not fully understood, or applicable. To test the applicability of the equations by Qatu for complex geometries, one needs to extend the equations by Qatu and develop the necessary set of equations for various geometries, specially those with a non-constant Lame parameter, like conical shells. The set of equations presented by Qatu can be applied directly to spherical, cylindrical and other shells of constant Lame parameters (Qatu, 1999).

The author of this reply sees that the need for higher order shear deformation theories have *not* yet been established. Good research needs to be done to show that such theories present closer results to those obtained using the three-dimensional theory of elasticity. Such higher order theories should follow the methodology by Qatu (1999) to obtain the needed accuracy. Furthermore, unless a simple and accurate theory is presented for shell structures, complex higher order shell theories often have the same degree of complexity of the equations used in the three-dimensional of elasticity that they are trying to approximate. This will make higher order theories less attractive.

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